

MID-SEMSTER EXAMINATION  
 B. Math III, I SEMESTER, 2013-14  
 PROBABILITY III

Max. marks: 100

Time limit: 2 1/2 hrs.

[In the problems below  $P$  stands for the transition matrix of a homogeneous Markov chain]

1. Let  $P = \begin{bmatrix} 1/3 & 2/3 \\ 1/4 & 3/4 \end{bmatrix}$ . Find  $P^n$  and  $\lim_{n \rightarrow \infty} P^n$  (if it exists). Find the mean return time to each state. [30]

2. If  $i$  is a recurrent state and  $i \rightarrow j$  show that  $j$  is recurrent and  $j \rightarrow i$ . [20]

3. Let  $P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1/4 & 1/2 & 1/4 & 0 & 0 & 0 \\ 0 & 1/5 & 2/5 & 2/5 & 0 & 0 \\ 0 & 0 & 0 & 1/6 & 1/3 & 1/2 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1/4 & 0 & 3/4 \end{bmatrix}$ . Find closed irreducible sets, transient states and absorption probabilities. [30]

4. Let  $P = \begin{bmatrix} 1/2 & 0 & 1/3 & 0 & 1/6 \\ 1/4 & 1/4 & 0 & 1/2 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 0 & 1/2 & 0 & 1/4 \end{bmatrix}$ . Find all left eigen-vectors and all stationary distributions. Also find all transient states and closed irreducible sets. [20]